2nd International GeoGebra Conference 2011
Hagenberg near Linz
Austria
Dynamic Illustrations Of Some Fibonacci Identities

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August 29, 2011

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Practica Geometriae (Practice of Geometry). Here he employed algebra to solve geometric problems and used geometry to solve algebraic problems, a radical approach for Europe for his day.

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This identity, known as **Cassini's identity** is the driving force behind Sam Lloyd's puzzle.

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Acknowledgements

I express my sincere thanks to Dr. I.K.Rana and Dr. S. Shirali for their relentless support in this project

References

A Primer On Number Sequences , Dr. Shailesh Shirali

Fibonacci And Lucas Numbers With Applications , Prof. Thomas Koshy